Knowledge Representation and Reasoning for Design of Spindle Units of Machine Tools Using Intuitionistic Fuzzy Generalized Nets

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Abstract
A new approach for knowledge representation and reasoning for calculation of power losses in the spindle bearings using Intuitionistic Fuzzy Generalized Nets is proposed in this paper. The knowledge is divided in two general categories: heuristic and model-based knowledge. The encoding of knowledge and the control structure are presented by Generalized sub-nets including different type of transition conditions and characteristic functions (crisp, fuzzy, intuitionistic fuzzy). The proposed system is tested for a great amount of practical examples and some results are presented and discussed in this paper.

1. Introduction
The rapidly development of computer technologies has allow in the last years to estimate much more behavior parameters of machine-building products even in the design stage and this way to perform preliminary design optimization. This approach leads to essential reduction of costs in time and labor for creating of new products.

In this paper is illustrated how the mathematical models can be integrated as a Knowledge Base (KB) to provide an intelligent working environment for the Design Engineer. Models are defined as functions of goals and knowledge [9]. In this multi model system different models encode different knowledge and most often this knowledge is from very different sources. The combination of uncertainty with knowledge based inference and large scale data storage make possible to estimate the values of missing data by comparison to similar cases in the DB and opens up new possibilities in the management of complex information.

The available information is in the form of tables of parametric values, compilations of experimental data, graphs and empirical correlations. In this large Database (DB) a part of information is incomplete or uncertain.

The main aim of present study is to offer an approach for knowledge representation and reasoning based on analytical and experimental models for calculation of power losses in the spindle bearings. The most KB systems are based on the rule-based approach by using production rules, semantic networks, frames etc. [10]. The present report leans on another representation approach namely that of Generalized nets (GNs). GNs are graphical and mathematical modeling tools, which consists of places, transitions and arcs that connect them [1, 2]. They are an extension of Petri nets and contain different logical operations, quantifiers and modal operators. GNs are a promising tool for describing and studying systems that are characterized as being concurrent, asynchronous, distributed, parallel and stochastic.

In this paper the main characteristics of GNs knowledge representation strategy and control structure (reasoning) are presented. The tools implementation is based on a great amount of practical examples and the results are precisely evaluated and analyzed.

2. Intuitionistic fuzzy generalized nets (IFGN)
The GNs are an appropriate tool for modeling of real events phenomenon and complex structures because of their flexibility and universal modeling possibilities. They allow description of dynamic and steady-state behavior of complex systems. The main advantage of using the GNs is the possibility for fast simulation of various systems with complex structure. It makes the GNs very suitable tool for modeling of information systems.

The use of Intuitionistic Fuzzy Sets for extending the ordinary GNs opens new directions for their implementation, i.e. to operate with uncertain and imprecise information in the GN-model.

In the definition of the concept GN [2] is noticed that the function \( f(.) \) that checks the truth values of the predicates can give different type values: in crisp GNs they are elements of the set \{“false”, “true”\}; in Fuzzy
GNs - it has values in interval [0,1]; in Intuitionistic Fuzzy GNs - it has values in set [0,1] x [0,1]. All other components, without the component \( F(.) \) and \( \Phi(.) \) are the same. The function \( \Phi(.) \) now gives to every token as current characteristic two values: the first coincides with the token characteristic in the sense of GNs; the second is an ordered tuple of real numbers, each one of which is a number within the range [0,1]. They are equal to the truth’s values of the predicate of the transition condition between the starting and destination place of the transfer. The function \( f(.) \) calculates these two values of corresponding predicates \( r_{ij} \) (1) and is defined by means of Intuitionistic Fuzzy Sets (IFS).

\[
f(r_{ij}) = \langle \mu(r_{ij}), v(r_{ij}) \rangle
\]

where \( \mu(r_{ij}) \) is the degree of truth of the predicate \( r_{ij} \) and \( v(r_{ij}) \) is its degree of false with the constraint (2)

\[
0 \leq \mu(r_{ij}) + v(r_{ij}) \leq 1
\]

The IFSs follow the basic sets and logic operations defined in the fuzzy sets. For the suggested approach the following operations over IFSs and Intuitionistic fuzzy rules are important:

- product over IFSs (3)

\[
P \cdot Q = \{ x, \mu_p(x). \mu_Q(x) + v_p(x) - v_Q(x), v_p(x) \} (3)
\]

- summation over IFSs (4)

\[
P + Q = \{ x, \mu_p(x) + \mu_Q(x) - \mu_p(x) \mu_Q(x), v_p(x) \} (4)
\]

- negation over IFSs and rules (5)

\[
f(\neg p) = \langle v(p), \mu(p) \rangle (5)
\]

- intersection over IFSs and rules (6)

\[
f(p \land q) = F(p) \cap F(q) = \\
= \langle \min(\mu_p, \mu_q), \max(v_p, v_q) \rangle
\]

- union over IFSs and rules (7)

\[
f(p \lor q) = F(p) \lor F(q) = \\
= \langle \max(\mu_p, \mu_q), \min(v_p, v_q) \rangle
\]

- max - min implication over rules (8)

\[
f(p \Rightarrow q) = F(p) \rightarrow F(q) = \\
= \langle \max(v_p, \mu_q), \min(\mu_p, v_q) \rangle
\]

Other type of Intuitionistic Fuzzy Generalized Nets is the second type (IFGNs). Here the tokens are some "quantities" which flow inside the net. The values of the transition condition's predicates can be intuitionistic fuzzy, i.e. they can have degrees of truth and of falsity. The capacities of the transition arcs described in index matrix \( M \) are real numbers. The essential difference between IFGNs and the other GNs is the set of the tokens \( K \) and the functions related to it. Now the elements of \( K \) are some "quantities" which have as an initial characteristic some "type" (element of the set \( X \)) and which do not receive other characteristics. The function \( \Theta_a \) gives the time - moment when a given token will enter the net as in the ordinary GNs. The function \( \Theta(.) \) has new meaning. Now it is related to the places, which receive function characteristics (the quantities of the tokens from the different times in the corresponding places).

In this work a combination of the described two types of IFGNs is used to develop the tools for knowledge representation and reasoning of spindle units design.

3. Mathematical models for calculation of power losses in different bearings

The modern design methods for machine tools require the application of different suitable modeling techniques and more over better models mean better designs. To achieve an optimal machine tool design it is necessary to evaluate the machine's thermal behaviors. A main node, which in the greater measure defines them, is namely the spindle unit. The heat generated in bearings causes considerable deformations, which influence on the accuracy of machine tools. Power losses in the bearing depend on the load and on several other factors, the most important of which are the bearing type and size, the operating speed, the properties and the quantity of the lubricant. They can be described analytically by models, known from the mechanic, which do not consider a set of designs features and so are in greater degrees approximate [3]. In many cases it is possible to use experimental received models, but their application is only within the framework of the concrete experimental conditions.

As a kernel of the proposed KB system are used 11 different models for calculating the power losses in the bearings, which are described and discussed in detail in [4, 5, 6, 7, 8].

The next part considers the main structure and the separated parts of the proposed system, based on the selected and analyzed models.

4. IFGNs for knowledge representation and reasoning

4.1. Global structure of the system

The proposed knowledge - based system, which global structure is shown on Fig. 1, can perform general symbolic and numerical calculations. The knowledge is divided in two general categories: heuristic knowledge (subnet \( E_i \)) and model - based knowledge (subnet \( E_j \)). This approach allows determining of the simplest and most appropriate representation scheme the interrelationship and interdependencies of the domain knowledge (objects, concepts and situations). The encoding of knowledge in machine-readable form is presented by Generalized subnet, which is highly suitable for encoding of design information. The rules are applied in chains of induction and deduction. The control strategy, which determines the order in which operations should be, performed use Meta knowledge to guide their inference. The Meta knowledge is presented by transition conditions which are index matrices including transition predicates (crisp, fuzzy or intuitionistic fuzzy) and the priorities of tokens, places and transitions. A best-first strategy of search and heuristic
evaluation function are used to accompany the chaining mechanism.

4.2. GN model for encoding the expert knowledge

The GN for encoding the expert knowledge is shown on Fig. 2 and is developed by using the graphical editor of the program system “GN”. The GN model includes 11 transitions and 26 places, from which 1 input and 2 outputs for the net. The token enter the place \( l_1 \) with no initial characteristics.

4.3. IFGN model for encoding of empirical and/or analytical knowledge

IFGN model for encoding of empirical and/or analytical knowledge \( (E_2) \) is shown on Fig 3. In difference to the presented above IFGN model, this model is from second type. In particular it concerns all transitions with the exception of the first and last one. The net includes 6 transitions and 25 places, from which – 2 input and 2 output places for the net.

The token enter the place \( l_1' \) with initial characteristics, received from the subnet for encoding the expert knowledge \( (E_1) \) (place \( l_{35} \)). The input place \( l_1' \) represent the connection of the net with the Date Base. The transitions have no time components at this stage of development. The output place \( l_{24}' \) represent the net connection with the subnet for the interface Mechanism \( (E_3) \), which use the token characteristics as initial characteristics.

Fig. 1 Global structure of the system

The subnet description is shown on Table 1. The first column of the table includes the mathematical definitions of transitions and transitions type. They represent the logical processes of selecting the bearing type \( (r_1) \) and features, on the basis of which the thermal behavior is investigated. The main bearing features covered by the firing the transitions are: definition of roller type \( (r_2) \), number of rows \( (r_3) \) and directions \( (r_4) \), the presence of self-aligning \( (r_5) \) and preload \( (r_6) \), the type of lubrication \( (r_7) \) and spindle units \( (r_9 - r_{11}) \) etc.

Table 2 show the description of transitions, their type and corresponding index matrices with their predicates. The first transition \( (r_1') \) represents the process of verification the correctness of initial characteristics of the tokens entering the input place. Transition \( r_2' \) describes the process of selecting the friction coefficient values from the DB for the specified case. For this purpose, the Tables 12a (for ball bearings) and 12b (for roller bearings) are used. The friction coefficient values are given by their lower and upper limit. The predicates are not precisely mathematical described only with the aim to simplify the presentation. The truths value of the predicates is evaluate by intuzionistic fuzzy function. The place capacity of \( l_5' \) is 1.

\( z_1' \) is similar to the above described, but in respect to fix the values of \( f_1 \) and \( f_0 \). The capacity of place \( l_9' \) is 2. Transitions \( z_2' \) and \( z_4' \) are identical to the described above, but they use other tables from the DB for determination of \( k_{fr}, k_p, l_p, Zp \) respectively.
4.4. IFGN model for Reasoning (Inference Mechanism)

The graphical structure of IFGN model for Reasoning – $E_r$, on the base of expert knowledge and rules and experimental, empirical and analytical models is presented in Fig. 4. The net for inference mechanism includes 8 transitions and 19 places, from which 1 input and 3 outputs in respect to the net.

Fig. 4 IFGN model for Reasoning

Table 3 shows the responding definitions of transitions, index matrices and their predicates in simplify form. For example the first transition ($z_1$) describes the process of evaluating the entering tokens for correctness. The index matrix includes 3 predicates, responding which truths value the tokens from the input place enter one or other output place and receive different complementary characteristics. All other transitions represent the process of decision making to apply different inference mechanisms (models) and this way they build the control structure of the system.

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5. Software implementation

The implementation of the proposed approach for knowledge representation and reasoning in design calculations of power losses was effected in collaboration with a MS-Office (especially MS Access 2000) and a software tool “GNET”. “GNET” is a software package for modeling and simulation using GNs and their extension. It provides an easy to use graphical interface for modeling behavior information and organization models. “GNET” are connected to MS-Office using DDE interface.

6. Conclusions

In this paper a new approach for knowledge base representation and reasoning is proposed. The developed information system uses the Generalized Nets to represent, exchange and share knowledge and data between three separated systems:

- Data Base for bearings catalog data;
- Set of GNs for description of analytical and experimental knowledge to calculate the thermal power loses at the spindle unit;
- Program system for Finite Element Method (FEM).

The suggested tool is applicable to many other cases of investigation and design of machine tools trough the FEM.

The future work is directed on building the interoperability between the design system and the system for applying the FEM.

References

4. Figatner А.М., (1987) An Influence of the preload of the roller bearings on capacity to work of spindle units of high exact tools, Stanki i instrument, № 2, pp. 10.
Table 1: Transitions description of $E_1$

<table>
<thead>
<tr>
<th>No</th>
<th>Transition / Transition Type</th>
<th>Index matrices</th>
<th>Predicates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$r_1 = (l_1, l_6, l_{12}, l_{13}, l_{23}, l_{30}, l_{34})$, $\Delta_1 = \lor(l_1, l_6, l_{12}, l_{13}, l_{23}, l_{30}, l_{34})$</td>
<td>$l_1 W_{27}, l_6 W_{28}$, $l_{12} W_{29}$, $l_{13} W_{30}$, $l_{23} W_{31}$, $l_{30} W_{32}$, $l_{34} W_{33}$</td>
<td>$W_{12} =$ &quot;Roller bearing&quot;</td>
</tr>
<tr>
<td>2</td>
<td>$r_2 = (l_2, l_6, l_{17}, l_8, l_{18}, l_{27}, l_{35})$, $\Delta_2 = \lor(l_2, l_6, l_{17}, l_8, l_{18}, l_{27}, l_{35})$</td>
<td>$l_2 W_{26}, l_8 W_{27}, l_{18} W_{28}$, $l_{35} W_{29}$</td>
<td>$W_{27} =$ &quot;Cylindrical roller&quot;</td>
</tr>
<tr>
<td>3</td>
<td>$r_3 = (l_3, l_{10}, l_{11}, l_{12}, l_3, l_{37})$, $\Delta_3 = \lor(l_3, l_{10}, l_{11}, l_{12}, l_3, l_{37})$</td>
<td>$l_3 W_{30}, l_{10} W_{31}$, $l_{11} W_{32}$</td>
<td>$W_{30} =$ &quot;Grease lubrication&quot;</td>
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<tr>
<td>4</td>
<td>$r_4 = (l_4, l_{18}, l_{19}, l_{20}, l_{22}, l_{23}, l_1)$, $\Delta_4 = \lor(l_4, l_{18}, l_{19}, l_{20}, l_{22}, l_{23}, l_1)$</td>
<td>$l_4 W_{33}, l_{18} W_{34}, l_{19} W_{35}$, $l_{20} W_{36}$, $l_{22} W_{37}$, $l_{23} W_{38}$, $l_1 W_{39}$</td>
<td>$W_{38} =$ &quot;Unidirectional bearing&quot;</td>
</tr>
<tr>
<td>5</td>
<td>$r_5 = (l_{14}, l_{15}, l_{16}, l_{17}, l_5, l_{37})$, $\Delta_5 = \lor(l_{14}, l_{15}, l_{16}, l_{17}, l_5, l_{37})$</td>
<td>$l_{14} W_{36}, l_{15} W_{37}$, $l_{16} W_{38}$, $l_{17} W_{39}$</td>
<td>$W_{39} =$ &quot;Self-aligning bearing&quot;</td>
</tr>
<tr>
<td>6</td>
<td>$r_6 = (l_{16}, l_{17}, l_8, l_{19}, l_6, l_{37})$, $\Delta_6 = \lor(l_{16}, l_{17}, l_8, l_{19}, l_6, l_{37})$</td>
<td>$l_{16} W_{40}, l_{17} W_{41}$, $l_{19} W_{42}$, $l_6 W_{43}$</td>
<td>$W_{43} =$ &quot;Cylindrical roller&quot;</td>
</tr>
<tr>
<td>7</td>
<td>$r_7 = (l_{18}, l_{19}, l_{20}, l_{22}, l_{23}, l_7, l_1)$, $\Delta_7 = \lor(l_{18}, l_{19}, l_{20}, l_{22}, l_{23}, l_7, l_1)$</td>
<td>$l_{18} W_{44}, l_{19} W_{45}, l_{20} W_{46}$, $l_{22} W_{47}$, $l_{23} W_{48}$, $l_7 W_{49}$, $l_1 W_{50}$</td>
<td>$W_{50} =$ &quot;Mixed lubrication&quot;</td>
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<tr>
<td>8</td>
<td>$r_8 = (l_{24}, l_{25}, l_{26}, l_{27}, l_8, l_5, l_{37})$, $\Delta_8 = \lor(l_{24}, l_{25}, l_{26}, l_{27}, l_8, l_5, l_{37})$</td>
<td>$l_{24} W_{51}, l_{25} W_{52}$, $l_{26} W_{53}$, $l_{27} W_{54}$, $l_8 W_{55}$, $l_5 W_{56}$, $l_{37} W_{57}$</td>
<td>$W_{57} =$ &quot;Mixed lubrication&quot;</td>
</tr>
<tr>
<td>9</td>
<td>$r_9 = (l_{24}, l_{25}, l_{26}, l_{27}, l_8, l_{13}, l_2, l_3)$, $\Delta_9 = \lor(l_{24}, l_{25}, l_{26}, l_{27}, l_8, l_{13}, l_2, l_3)$</td>
<td>$l_{24} W_{58}$, $l_{25} W_{59}$, $l_{26} W_{60}$, $l_{27} W_{61}$, $l_8 W_{62}$, $l_{13} W_{63}$, $l_2 W_{64}$, $l_3 W_{65}$</td>
<td>$W_{65} =$ &quot;Cylindrical roller&quot;</td>
</tr>
<tr>
<td>10</td>
<td>$r_{10} = (l_4, l_{27}, l_{28}, l_{30}, l_6, l_{37})$</td>
<td>$l_4 W_{66}$, $l_{27} W_{67}$, $l_{28} W_{68}$, $l_{30} W_{69}$</td>
<td>$W_{69} =$ &quot;Spindle unit NN3020k/SP + 234420SP&quot;</td>
</tr>
</tbody>
</table>
| 11 | $r_{11} = (l_{27}, l_{31}, l_{32}, l_{33}, l_{34}, l_{35}, l_1)$, $\Delta_{11} = \lor(l_{27}, l_{31}, l_{32}, l_{33}, l_{34}, l_{35}, l_1)$ | $l_{27} W_{70}$, $l_{31} W_{71}$, $l_{32} W_{72}$, $l_{33} W_{73}$, $l_{34} W_{74}$, $l_{35} W_{75}$, $l_1 W_{76}$ | $W_{76} =$ "Spindle unit B7016.TPA.P4.UL"
Table 2: Transitions description of $E_2$

<table>
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<th>No</th>
<th>Transition / Transition Type</th>
<th>Index matrices</th>
<th>Predicates</th>
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<tbody>
<tr>
<td>1</td>
<td>$\rho_1'=&lt;d_1', l_9', l_{14}', l_{19}', l_{23}', l_{25}'&gt;,&lt;l_2', l_3', z_1', \Delta_1'&gt;$</td>
<td>$l_1' = l_2'$, $l_7' = l_9'$, $l_5' = l_{14}'$, $l_{17}' = l_{19}'$, $l_{21}' = l_{23}'$, $l_{25}' = l_{25}'$</td>
<td>$W_{13} = \text{&quot;Data are correct&quot;}$, $W_{15} = \text{&quot;Model-2&quot;}$</td>
</tr>
<tr>
<td>2</td>
<td>$\rho_2'=&lt;d_3', l_5', l_{10}', l_{15}', l_{16}', l_{17}', l_{19}', l_{21}', l_{23}', l_{25}'&gt;$, $z_2', \Delta_2&gt;$</td>
<td>$l_1' = l_2'$, $l_7' = l_9'$, $l_5' = l_{14}'$, $l_{17}' = l_{19}'$, $l_{21}' = l_{23}'$, $l_{25}' = l_{25}'$</td>
<td>$W_{26} = \text{&quot;Fuzzy Values&quot;}$, $W_{27} = \text{&quot;Model-3&quot;}$</td>
</tr>
<tr>
<td>3</td>
<td>$\rho_3'=&lt;d_7', l_9', l_{10}', l_{15}', l_{16}', l_{17}', l_{19}', l_{21}', l_{23}', l_{25}'&gt;$, $z_3', \Delta_3&gt;$</td>
<td>$l_1' = l_2'$, $l_7' = l_9'$, $l_5' = l_{14}'$, $l_{17}' = l_{19}'$, $l_{21}' = l_{23}'$, $l_{25}' = l_{25}'$</td>
<td>$W_{31} = \text{&quot;Fuzzy Values&quot;}$, $W_{32} = \text{&quot;Model-4&quot;}$</td>
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<td>4</td>
<td>$\rho_4'=&lt;d_9', l_5', l_{10}', l_{15}', l_{16}', l_{17}', l_{19}', l_{21}', l_{23}', l_{25}'&gt;$, $z_4', \Delta_4&gt;$</td>
<td>$l_1' = l_2'$, $l_7' = l_9'$, $l_5' = l_{14}'$, $l_{17}' = l_{19}'$, $l_{21}' = l_{23}'$, $l_{25}' = l_{25}'$</td>
<td>$W_{36} = \text{&quot;Fuzzy Values&quot;}$, $W_{37} = \text{&quot;Model-5&quot;}$</td>
</tr>
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<td>5</td>
<td>$\rho_5'=&lt;d_1', l_9', l_{14}', l_{19}', l_{23}', l_{25}'&gt;,&lt;l_2', l_3', z_5', \Delta_5'&gt;$</td>
<td>$l_1' = l_2'$, $l_7' = l_9'$, $l_5' = l_{14}'$, $l_{17}' = l_{19}'$, $l_{21}' = l_{23}'$, $l_{25}' = l_{25}'$</td>
<td>$W_{41} = \text{&quot;Fuzzy Values&quot;}$, $W_{42} = \text{&quot;Model-6&quot;}$</td>
</tr>
<tr>
<td>6</td>
<td>$\rho_6'=&lt;d_2', l_9', l_{14}', l_{19}', l_{23}', l_{25}'&gt;, &lt;l_2', l_3', z_6', \Delta_6'&gt;$</td>
<td>$l_1' = l_2'$, $l_7' = l_9'$, $l_5' = l_{14}'$, $l_{17}' = l_{19}'$, $l_{21}' = l_{23}'$, $l_{25}' = l_{25}'$</td>
<td>$W_{46} = \text{&quot;Fuzzy Values&quot;}$, $W_{47} = \text{&quot;Model-7&quot;}$</td>
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</table>

Table 3: Transitions description of $E_3$

<table>
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<th>Predicates</th>
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<tbody>
<tr>
<td>1</td>
<td>$\rho_1'=&lt;d_1', l_9', l_{14}', l_{19}', l_{23}', l_{25}'&gt;,&lt;l_2', l_3', z_1', \Delta_1'&gt;$</td>
<td>$l_1' = l_2'$, $l_7' = l_9'$, $l_5' = l_{14}'$, $l_{17}' = l_{19}'$, $l_{21}' = l_{23}'$, $l_{25}' = l_{25}'$</td>
<td>$W_{13} = \text{&quot;Data are correct&quot;}$, $W_{15} = \text{&quot;Model-2&quot;}$</td>
</tr>
<tr>
<td>2</td>
<td>$\rho_2'=&lt;d_2', l_9', l_{14}', l_{19}', l_{23}', l_{25}'&gt;, &lt;l_2', l_3', z_2', \Delta_2'&gt;$</td>
<td>$l_1' = l_2'$, $l_7' = l_9'$, $l_5' = l_{14}'$, $l_{17}' = l_{19}'$, $l_{21}' = l_{23}'$, $l_{25}' = l_{25}'$</td>
<td>$W_{26} = \text{&quot;Fuzzy Values&quot;}$, $W_{27} = \text{&quot;Model-3&quot;}$</td>
</tr>
<tr>
<td>3</td>
<td>$\rho_3'=&lt;d_9', l_9', l_{10}', l_{15}', l_{16}', l_{17}', l_{19}', l_{21}', l_{23}', l_{25}'&gt;, &lt;l_2', l_3', z_3', \Delta_3'&gt;$</td>
<td>$l_1' = l_2'$, $l_7' = l_9'$, $l_5' = l_{14}'$, $l_{17}' = l_{19}'$, $l_{21}' = l_{23}'$, $l_{25}' = l_{25}'$</td>
<td>$W_{31} = \text{&quot;Fuzzy Values&quot;}$, $W_{32} = \text{&quot;Model-4&quot;}$</td>
</tr>
<tr>
<td>4</td>
<td>$\rho_4'=&lt;d_9', l_9', l_{10}', l_{15}', l_{16}', l_{17}', l_{19}', l_{21}', l_{23}', l_{25}'&gt;, &lt;l_2', l_3', z_4', \Delta_4'&gt;$</td>
<td>$l_1' = l_2'$, $l_7' = l_9'$, $l_5' = l_{14}'$, $l_{17}' = l_{19}'$, $l_{21}' = l_{23}'$, $l_{25}' = l_{25}'$</td>
<td>$W_{36} = \text{&quot;Fuzzy Values&quot;}$, $W_{37} = \text{&quot;Model-5&quot;}$</td>
</tr>
<tr>
<td>5</td>
<td>$\rho_5'=&lt;d_1', l_9', l_{14}', l_{19}', l_{23}', l_{25}'&gt;,&lt;l_2', l_3', z_5', \Delta_5'&gt;$</td>
<td>$l_1' = l_2'$, $l_7' = l_9'$, $l_5' = l_{14}'$, $l_{17}' = l_{19}'$, $l_{21}' = l_{23}'$, $l_{25}' = l_{25}'$</td>
<td>$W_{41} = \text{&quot;Fuzzy Values&quot;}$, $W_{42} = \text{&quot;Model-6&quot;}$</td>
</tr>
<tr>
<td>6</td>
<td>$\rho_6'=&lt;d_2', l_9', l_{14}', l_{19}', l_{23}', l_{25}'&gt;, &lt;l_2', l_3', z_6', \Delta_6'&gt;$</td>
<td>$l_1' = l_2'$, $l_7' = l_9'$, $l_5' = l_{14}'$, $l_{17}' = l_{19}'$, $l_{21}' = l_{23}'$, $l_{25}' = l_{25}'$</td>
<td>$W_{46} = \text{&quot;Fuzzy Values&quot;}$, $W_{47} = \text{&quot;Model-7&quot;}$</td>
</tr>
</tbody>
</table>

Knowledge Representation and Reasoning for Design of Spindle Units of Machine Tools Using Intuitionistic Fuzzy Generalized Nets